Anisotropic Reversible Permeability Effects in the Magnetic Reproduce Process

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Abstract—The effect of an anisotropic, reversible medium permeability on the reproduce flux from an arbitrary recorded magnetization pattern has been calculated. It is shown that for normally oriented media the anisotropy of the permeability always acts to increase the reproduce flux. A 3-5 dB increase in short wavelength sensitivity is predicted in ac-biased recording due to this reproduce phenomenon. At short wavelengths, the effect of head-to-medium spacing is shown to assume a general form independent of the recorded magnetization pattern and dependent only on the geometric mean of the two orthogonal permeabilities. For short wavelengths and small head-to-medium spacings, a loss of approximately exp $(-4\pi a/\lambda)$ (110 a/λ [dB]) is predicted for most media where a is the head-to-medium spacing and λ is the wavelength.

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Introduction

Theoretical analysis of the magnetic reproducing process is relatively straightforward. At each instant it suffices to determine the flux threading the reproduce head windings due to a recorded magnetic medium passing the reproduce head. Head materials generally exhibit sufficiently high permeability so that it is possible to separate pure frequency effects from

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those dependent solely on the wavelength or spatial variation of the magnetization. The frequency effects are associated with the efficiency of flux transfer from the head surface to the reproduce windings, whereas the wavelength dependence of the flux is affected by such factors as external head geometry, head-to-medium positioning, and medium permeability which control the flux impinging the reproduce head surface due to the recorded magnetization pattern. In this study, the effect of an anisotropic, reversible medium permeability on the reproduce wavelength response in magnetic recording has been calculated.

The wavelength response of a recorded magnetic medium exhibiting an isotropic reversible permeability has been analyzed by Westmijze [1]. He considered a medium magnetized uniformly through its depth and sinusoidal in the recording direction. Both longitudinal and vertical recorded magnetization directions were examined. The reproduce head was taken to be a semi-infinite plane so that the effects of gap length, core length, and track width on the response were not included. Westmijze's essential conclusion was that at short wavelengths, an isotropic permeability reduced the reproduce flux but only under the condition of a finite head-to-medium spacing.

Most high quality recording media are composed of highly oriented single domain particles and thus exhibit magnetic behavior which is highly anisotropic. Typically, a medium well-oriented in the head-to-medium motion direction possesses a relatively large reversible permeability (3-5) in the vertical direction and nearly unit permeability (1-2) in the orientation direction. Thus it is reasonable to examine the effect of an anisotropic reversible permeability on the wavelength response. In the analysis presented here, a magnetization pattern of arbitrary depth variation and direction is assumed recorded in the medium. The reversible permeability is taken to be anisotropic and expressible by a diagonal tensor. In the following section, the essentials of the calculation are presented. The solution requires a knowledge of the head field altered by the anisotropic medium. In the Appendix, this field is determined by the use of Green's functions. In the third section, the expression for the reproduce flux is given in complete form and is examined for general behavior as well as evaluated for recording magnetization patterns specific to both biased and unbiased recording. In contrast to the effect of an isotropic permeability, the anisotropy of the permeability acts to increase the reproduce flux for media oriented and magnetized in the head-to-medium motion direction. With head-to-medium spacing, the flux is reduced by the geometric mean of the two orthogonal permeabilities. As part of a study of anhysteretic contact duplication, Tjaden and Rijckaert have also computed, but without interpretation, the general flux response [2]. Here, however, an attempt is made to present the results (and derivation) in a particularly transparent form, and in particular, emphasis is placed upon physical interpretation of the phenomena.

Theory

It is assumed that the magnetic medium has been recorded with a permanent remanent vector magnetization $M_0(r)$. Thus the net magnetization in the medium after leaving the influence of the record head fields is given by

$$M(r) = M_0(r) + \widetilde{\chi} \cdot H_m(r) \tag{1}$$

where $H_m(r)$ is the demagnetization field at each point in the medium and $\widetilde{\chi}$ is a tensor susceptibility. For this analysis $\widetilde{\chi}$ will be approximated by a constant diagonal matrix. The reversible (relative) permeability is then given by

$$\widetilde{\mu} = \widetilde{1} + \widetilde{\chi} = \begin{pmatrix} \mu_x & 0 \\ 0 & \mu_y \end{pmatrix} \tag{2}$$

where μ_x, μ_y are the permeabilities measured in the recording plane with x denoting the head-to-medium motion direction while y represents the vertical to the medium plane. The constitutive relation may be written, using (1) and (2), as

$$B(r) = \mu_0(\widetilde{\mu} \cdot H_m(r) + M_0(r)) \tag{3}$$

where $\mu_0 = 4\pi \times 10^{-7}$ H/m is the permeability of free space and $B(r)$ is the induction.

The flux impinging the reproduce head may be computed by either of three ways. First, the normal component of $B(r)$ may be determined at the surface of the reproduce head and integrated to find the net flux. Second, utilizing reciprocity, the flux may be computed from the relation

$$\phi(t) = \mu_0 \int dv\, H_0(r) \cdot M(r, t) \tag{4}$$

where $H_0(r)$ is the field per unit MMF due to the reproduce head unaffected by the presence of the permeable medium. Evaluation of (4) involves the computation of $H_m(r)$ in order to determine $M(r)$ via (1). A third alternative involves a different form for the reciprocity relation

$$\phi(t) = \mu_0 \int dv\, H(r) \cdot M_0(r, t) \tag{5}$$

where $H(r)$ is the field per unit MMF of the reproduce head in the presence of the permeable medium [3]. Equation (5) gives the flux in a particularly physical representation since, given a recorded remanent magnetization $M_0(r)$, the spatial sampling by the reproduce head of the recorded magnetization can be seen directly.

In this calculation, cross track invariance will be assumed so that a two-dimensional analysis will be utilized and the resultant flux will be proportional to the reproduce head track width w. In addition, since the wavelength response is usually of interest, the recorded magnetization will be described by its Fourier components (lower case denotes the Fourier transform of an upper case variable).

$$m_0(k, y) = \int_{-\infty}^{\infty} dx\, e^{-ikx} M_0(x, y)$$

$$M_0(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk\, e^{ikx} m_0(k, y) \tag{6}$$

where $k = 2\pi/\lambda$ is the wavenumber and λ is the wavelength. Similarly, the wavelength response of the flux, from (5), (and

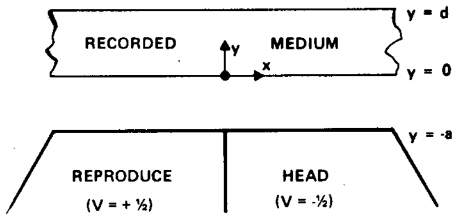

Fig. 1. Two-dimensional geometry for the reproduce flux calculation. *d* represents the depth of recording which may be less than the coating thickness. Boundary potentials on the head surface are indicated for derivation of field per unit MMF.

Fig. 1) is

$$\phi(k) = \mu_0 w \int_0^d dy\, \boldsymbol{h}^*(k, y) \cdot \boldsymbol{m}_0(k, y) \tag{7}$$

where $\boldsymbol{h}^*(k, y)$ is the complex conjugate of the Fourier component of the head field per unit magnetomotive force.

The field $H(r)$ may be determined by solving an anisotropic Laplace equation for the potential

$$\nabla \cdot \tilde{\boldsymbol{\mu}} \cdot \nabla V(r) = 0 \tag{8}$$

with the potentials on the head surface set to give a unit MMF drop across the gap (Fig. 1), and where

$$\boldsymbol{H}(r) = -\nabla V(r). \tag{9}$$

In two dimensions, (8) becomes

$$\mu_y \frac{\partial^2}{\partial y^2} V(r) + \mu_x \frac{\partial^2}{\partial x^2} V(r) = 0 \tag{10}$$

and Fourier transforming yields

$$\frac{\partial^2}{\partial y^2} v(k, y) - k^2 \frac{\mu_x}{\mu_y} v(k, y) = 0. \tag{11}$$

The solution is of the form

$$v(k, y) \propto e^{\pm k\sqrt{(\mu_x/\mu_y)}y} \tag{12}$$

so that

$$\boldsymbol{h}(k, y) \propto e^{\pm k\sqrt{(\mu_x/\mu_y)}y}. \tag{13}$$

The essence of the effect of anisotropy on the reproduce flux can be seen in (13). For anisotropic or oriented media, the ratio $\sqrt{\mu_x/\mu_y}$ is not unity. This anisotropy factor scales with the wavenumber k in (13) so that the spatial decay of the head field or, equivalently, the effective wavelength is altered by this anisotropy factor. In particular, for the typical case of an in-plane oriented medium for which $\mu_x < \mu_y$, the anisotropy factor is less than unity. In this instance, the effective wavelength $(\lambda\sqrt{\mu_y/\mu_x})$ is lengthened so that the reproduce head field "looks deeper" into the tape. A decreasing anisotropy factor thus gives less spatial decay of the field and hence more reproduce flux via (7).

The analysis leading to $v(k, y)$ and $h(k, y)$ is given in the Appendix for the case of a reproduce head with negligibly small gap length and infinitely long core length. Utilizing $h(k, y)$ with (7) yields the reproduce flux response which is given in the next section.

Results and Discussion

The Fourier component of the reproduce head flux or "wavelength response" is given by evaluating (7) with the field given by (A-7), (A-9), (A-10), and (A-12) in the Appendix. The result is

$$\begin{aligned}\phi(k) = \mu_0 w \Big\{ &(\beta + \tanh k\alpha d) \int_0^d dy' [m_x(k, y') \cosh k\alpha y' \\ &- (\alpha/i)\, m_y(k, y') \sinh k\alpha y'] + (1 + \beta \tanh k\alpha d) \\ &\cdot \int_0^d dy' [(\alpha/i)\, m_y(k, y') \cosh k\alpha y' - m_x(k, y') \\ &\cdot \sinh k\alpha y'] \Big\} \Big/ \{\cosh ka[\beta(1 + \tanh ka) \\ &+ (\beta^2 \tanh ka + 1) \tanh k\alpha d]\},\end{aligned} \tag{14}$$

where

- w track width,
- d recording depth,
- a head-to-medium spacing,
- k wavenumber (2π/wavelength),
- μ_x reversible relative permeability in the head-to-medium motion direction,
- μ_y reversible relative permeability in the direction vertical to the medium plane,
- α = $\sqrt{\mu_x/\mu_y}$ = anisotropy factor,
- β = $\sqrt{\mu_x \mu_y}$ = mean permeability,
- $m_x(k, y)$ kth Fourier component of the recorded remanent longitudinal magnetization pattern,
- $m_y(k, y)$ kth Fourier component of the recorded remanent vertical magnetization pattern.

Phase information is contained in the possibly complex $m_x(k, y)$, $m_y(k, y)$: the $i = \sqrt{-1}$ term indicates the 90° reproduce phase shift between these two orthogonal directions of magnetization. In this section, $m_x(k, y)$, $m_y(k, y)$ always refer to components of the remanent magnetization; for convenience, the "zero subscript" notation utilized in the previous section is deleted.

First some general properties of (14) are examined before evaluating the flux for specific magnetization patterns. Most recording media consist of well-oriented particles in the head-to-medium motion direction. In such cases, the vertical permeability μ_y, is much larger than the permeability in the oriented direction μ_x. Thus the anisotropy factor α will be less than unity, and as mentioned in the second section, the "effective wavelength" (λ/α) will be increased. In fact, in the limit of very large μ_y ($\alpha \to 0$, $\beta \to \infty$, $\alpha\beta = \mu_x$), the reproduce flux is

$$\phi(k) = \mu_0 w e^{-ka} \int_0^d m_x(k, y') dy'. \tag{15}$$

Thus, except for the effect of head-to-medium spacing, the "effective wavelength" in the medium becomes extremely long so that the total flux is reproduced. No contribution from vertical recorded magnetization occurs since lengthening the

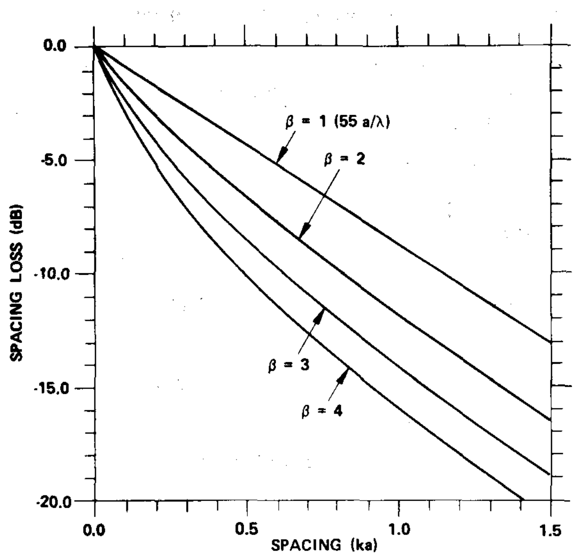

Fig. 2. Effect of head-to-medium spacing on the reproduce flux. Spacing loss (dB) is plotted versus normalized spacing ka where $k = 2\pi/\lambda$; λ is the wavelength and a is head-to-medium spacing. β is the mean geometric permeability.

"reproduce wavelength" by increasing μ_y also demagnetizes $m_y(k, y)$. Thus, in the usual case of oriented media with an in-plane recorded magnetization $m_x(k, y)$, the result of increasing μ_y increases the reproduce flux limited only by (15).

It is interesting to examine the reproduce flux for the case of wavelengths short with respect to the recording depth ($k\alpha d \gg 1$). In this case (14) reduces to

$$\phi(k) = \mu_0 w \left\{ \frac{2e^{-ka}}{\beta + 1 - (\beta - 1)e^{-2ka}} \right\} \int_0^d dy' e^{-k\alpha y'} \cdot [m_x(k, y') + (\alpha/i)\, m_y(k, y')]. \quad (16)$$

The effect of head-to-medium spacing is expressed by the bracketed factor in front of the integral. It solely depends upon the geometric mean of the permeability β and is independent of the particular recorded magnetization pattern. This factor is plotted in Fig. 2 in the form of flux level (dB) versus normalized spacing (ka) for several values of the mean permeability β. In the case of unit permeability ($\beta = 1$), the well-known exponential loss applies ($55a/\lambda$ [dB]). For a mean permeability greater than unity, the loss is pronounced at small head to tape spacings; at large spacings, the loss slope follows the unit permeability case but with a fixed offset. This behavior is summarized by

$$\phi(k) \propto e^{-\beta ka} \; (55\,\beta a/\lambda \text{ [dB]}), \qquad ka \ll 1$$

$$\phi(k) \propto \frac{2}{\beta + 1} e^{-ka}, \qquad ka \gg 1. \quad (17)$$

Thus for most media where typically $\beta \cong 2$, the initial loss slope is about 110 a/λ [dB] or twice the usual expression, and the large spacing offset is about 3.3 dB. For isotropic media ($\beta = \mu$), the spacing loss dependence is identical to that calculated by Westmijze for his specific magnetization distributions.

In order to examine the effect of the anisotropy on the wavelength response, it is necessary to specify the magnetization distribution. Here four cases are considered: 1) a sinusoidal magnetization pattern of wavenumber k directed along the head-to-medium motion direction with no variation through the depth; 2), as in 1), but with a vertically directed magnetization; 3), as in 1), but with an amplitude linearly increasing with depth into the medium (ac biased recording); and 4) alternating transitions of magnetization directed as in 1) and 3) with a transition parameter which increases linearly with depth into the medium (unbiased square wave recording). For convenience $\phi_0 = \mu_0 w M_0 d$ denotes the total flux.

Case 1:

$$m_x(k, y') = M_0, \qquad m_y(k, y') = 0$$

$$\phi(k)/\phi_0 = \frac{\tanh k\alpha d}{k\alpha d} \cdot \frac{\beta + \tanh k\alpha d/2}{\cosh ka\,[\beta(1 + \tanh ka) + (\beta^2 \tanh ka + 1)\tanh k\alpha d]} \quad (18)$$

which yields at short wavelengths and no head-to-medium spacing

$$\phi(k)/\phi_0 = \frac{1}{kd\alpha} = \frac{1}{kd}\sqrt{\frac{\mu_y}{\mu_x}}, \qquad k\alpha d \gg 1, a = 0. \quad (19)$$

The notable behavior of the reproduce flux at short wavelengths (19) is its dependence on the anisotropy of the permeability. In the case of "normal recording" when a longitudinal magnetization predominates, the reproduce flux is increased due to medium orientation in the recording direction ($\mu_y \gg \mu_x$). The reason for this behavior can be seen from the demagnetization fields at the surface of the head before reversible permeability effects occur. In the presence of the high permeability reproduce head, the longitudinal field increases from zero at the surface to a value equal to the remanent magnetization, while the vertical field decreases from the same maximum to zero in the interior. Nevertheless the exponential weighted averages (by the reproduce head) are the same. Therefore, for $\mu_y > \mu_x$, the increase in flux due to an induced vertical magnetization by μ_y is greater than the decrease due to magnetization by μ_x. With head-to-medium spacing, this increase is counteracted by the effect of the mean permeability (16) so that the net effect on the flux depends on the magnitudes of α, β, and ka. The isotropic case considered by Westmijze follows directly from (18) with $\alpha = 1$, $\beta = \mu$. Thus via (16) and (19), an isotropic permeability affects the reproduce flux only when a head-to-medium spacing occurs.

Case 2:

$$m_x(k, y') = 0, \qquad m_y(k, y') = M_0$$

$$\phi(k)/\phi_0 = \frac{\tanh k\alpha d}{kd} \cdot \frac{(1 + \beta \tanh k\alpha d/2)}{\cosh ka\,[\beta(1 + \tanh ka) + (\beta^2 \tanh ka + 1)\tanh k\alpha d]} \quad (20)$$

which yields with no head-to-medium spacing

$$\phi(k)/\phi_0 = \begin{cases} \dfrac{1}{kd}, & k\alpha d \gg 1 \\ \dfrac{1}{\beta} = \dfrac{1}{\sqrt{\mu_x \mu_y}}, & k\alpha d \ll 1. \end{cases} \tag{21}$$

In this case of uniform vertical recorded magnetization, the short wavelength limit is unaffected by the anisotropy of the permeability. The reason is that at short wavelengths and no spacing neither an in-plane nor vertical demagnetization field occurs at the surface in the medium. Conversely, as expected, the long wavelength component is demagnetized, and for anisotropic medium, by the mean of the permeability β. This case was also considered by Westmijze whose result follows from (20) with $\alpha = 1, \beta = \mu$.

The previous two cases were considered primarily as simple illustrative examples of the effect of an anisotropic permeability. In the following two cases patterns will be examined which adhere to current ideas of ac-biased and unbiased recording.

Case 3:

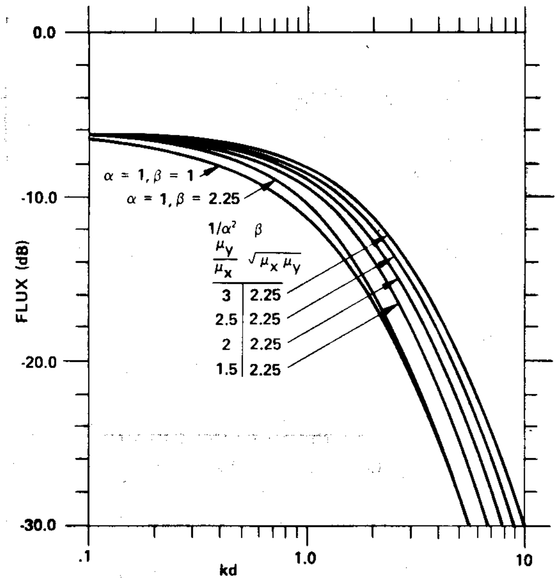

Fig. 3. Flux response level (dB) versus normalized wavenumber kd for ac biased recording. $\beta = 2.25$ represents a mean permeability typical for many recording materials.

$$m_x(k, y') = M_0 y'/d, \qquad m_y(k, y') = 0$$

$$\phi(k)/\phi_0 = \frac{[\beta + \tanh k\alpha d - (\beta + k\alpha d)\,\mathrm{sech}\, k\alpha d]}{(k\alpha d)^2 \cosh ka[\beta(1 + \tanh ka) + (\beta^2 \tanh ka + 1)\tanh k\alpha d]} \tag{22}$$

and in the limit of short wavelengths ($k\alpha d \gg 1$) and no head-to-medium space ($a = 0$):

$$\phi(k)/\phi_0 = \frac{1}{(kd\alpha)^2} = \frac{1}{(kd)^2}\frac{\mu_y}{\mu_x}. \tag{23}$$

It has been shown that an in-plane magnetization which varies linearly with depth into the tape is a reasonable model for ac biased recording [4]. In this case, M_0 denotes the product of the anhysteretic susceptibility times the signal field. The flux wavelength response (22) shows a large dependence on the anisotropy ratio at a short wavelength (23). In Fig. 3, flux response is plotted for various examples of mean permeability β and anisotropy ratio α. For comparison, the unit permeability case ($\alpha = \beta = 1$) is plotted as well as an example of an isotropic medium ($\alpha = 1, \beta = 2.25$). The value of β chosen is typical of present media and is relatively insensitive to the degree of orientation. It is interesting that the effect of an anisotropic permeability is to increase the short wavelength response approximately 6 dB for a well-oriented medium corresponding to ($\mu_y/\mu_x = 2$).

Case 4:

$$m_x(k, y') = \frac{4}{\pi} M_0 e^{-k(a_s y' + a_f)}, \qquad m_y(k, y') = 0,$$

$$a_s = \frac{a_b - a_f}{d}$$

$$\phi(k)/\phi_0 = \frac{\dfrac{4}{\pi} e^{-ka_f}\left[\dfrac{\beta + 1}{a_s \pm \alpha}\right][e^{\pm\alpha kd} - e^{-ka_s d}]}{2\,kd \cosh ka \cosh k\alpha d\,[\beta \tanh ka + (\beta^2 \tanh ka + 1)\tanh k\alpha d]} \tag{24}$$

and in the limit of short wavelengths ($k\alpha d \gg 1$) and no head-to-medium space ($a = 0$):

$$\phi(k)/\phi_0 = \frac{e^{-ka_f}}{kd(\alpha + a_s)}. \tag{25}$$

The ± means the total flux is actually the sum of (24) over the two possible sign terms. In this case, $m_x(k, y')$ represents the fundamental component of a series of alternating transitions at wavenumber k. The form of $m_x(k, y')$ derives from assuming that each transition of magnetization is represented by an arctangent whose transition parameter increases linearly with depth into the tape. This appears to be a reasonably good model of unbiased recording for thick media in which the record current is set to maximize the short wavelength flux [5]. In (24), d is again the depth of recording, a_f is the transition parameter at the front surface, and a_b is the transition parameter at the back of the recorded layer ($y = d$).

In Fig. 4, head replay voltage ($k\phi$) is plotted for the case of no head-to-medium spacing, a surface transition parameter of one-third the penetration depth and a back parameter equal to

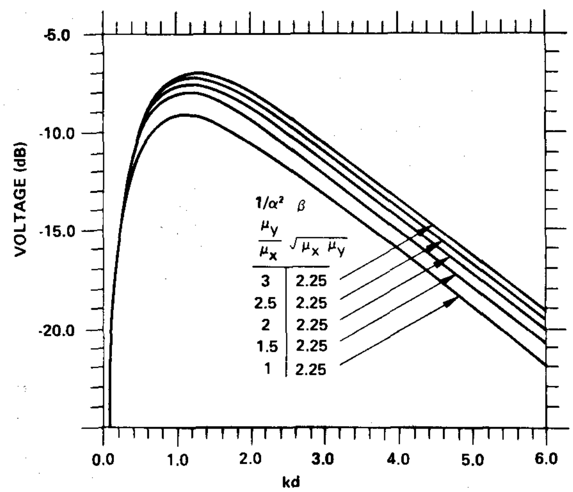

Fig. 4. Voltage response level (dB) versus normalized wavenumber kd for unbiased recording. $k = 2\pi/\lambda$, λ is the wavelength, d is the depth of recording. $\beta = 2.25$ is a typical mean permeability.

the penetration depth. These parameters seem representative of unbiased short wavelength optimized, square-wave recording. Voltage is plotted since actual systems (such as video recorders) do not integrate the replay voltage to obtain flux. According to (25), only the anisotropy of the permeability affects the short wavelength output (for $a = 0$). Again, as in cases 1 and 3, normal particle orientation (decreasing α) yields, with longitudinally directed magnetization, increased replay voltage. However, an interesting comparison between cases 1, 3, and 4 is that the increase is weakest for a recorded magnetization which decreases with depth into the medium (this case) and is strongest for a recorded pattern with increases with recording depth (case 3).

Throughout this discussion, the effect of the mean permeability which enters through the head-to-medium spacing has been discussed separately from the effect of the anisotropy of the permeability. For a given head-to-medium spacing the effect, conceptually, of developing a reversible anisotropic permeability depends on the relative magnitudes of (16) and (12), (21), (23), or (25) given the specific magnetization pattern. It is possible with a sufficient head-to-medium spacing (or ka) that the net flux may decrease. In such a case, the increase due to the anisotropy of the permeability is offset by the decrease due to the mean permeability.

It is possible to show that the reversible magnetization changes which occur due to the demagnetization fields should not lead either to irreversible demagnetization for the magnetization in the recorded direction or saturation in the vertical direction in the case of short wavelength saturation recording (case 4). At short wavelengths and no head-to-medium spacing (24) becomes

$$\phi = \frac{4}{\pi} \frac{M_r w \mu_0 e^{-ka_f}}{k(\alpha + a_s)} \tag{26}$$

The vertical component of the field at the head surface is, therefore,

$$H_y = \frac{k\phi}{\mu_0 w} = \frac{4}{\pi} \frac{M_r e^{-ka_f}}{(\alpha + a_s)}. \tag{27}$$

Thus, the magnetization at the surface is

$$M_y = \frac{\mu_y - 1}{\mu_y} \frac{4}{\pi} \frac{M_r e^{-ka_f}}{\alpha + a_s}. \tag{28}$$

Using typical values for the constants quoted earlier (and $\lambda < 6a_f$) insures in this case that saturation is not reached. Further, while the medium is in free space away from either the recording or reproducing head, the surface longitudinal field is from (27) and extrapolation of (16):

$$\frac{H_x}{H_c} = \frac{4}{\pi} \frac{M_r}{H_c} \frac{e^{-ka_f}}{(\alpha + a_s)(\beta + 1)}, \tag{29}$$

and again with reasonable estimates, for this example, the coercivity is not exceeded. This example illustrates how the limits of reversibility may be tested. Whether this limit is exceeded or not depends not only on the material constants but the presumed recorded magnetization distribution.

For consistency, it is worthwhile to note that if the replay head were just a wire (the so-called "open circuit" flux), the reproduce flux would be given by

$$\phi_{oc}(k) = e^{-ka} \lim_{a \to \infty} \phi(k) \cosh ka \tag{30}$$

where $\phi(k)$ is the general "short circuit" flux given by (14).

It is to be emphasized that this study applies only to the reproduce process in magnetic recording. A recorded remanent magnetization is assumed in the medium, and the reproduce flux is calculated under the influence of an anisotropic medium permeability. Thus the question of how the medium permeability effects magnetic recording has not been answered completely. To consider the total record-reproduce process, the effect of medium permeability on the record process must be determined. This is difficult since recording is a complicated, and in general, nonlinear problem. It is entirely possible, for example, that the rather large increase in short wavelength sensitivity in ac-biased recording predicted here (Case 3) may be reduced somewhat by compensating effects during the record process.

Conclusion

The influence of an anisotropic medium permeability on the reproduce process in magnetic recording has been determined. The notable results are, at short wavelengths, an increase in output due to the anisotropy of the permeability (medium orientation) and an increased spacing loss due to the geometric mean of the tensor permeability. The effect of the reproduce gap length on the reproduce response of an anisotropic permeable recorded medium has not been considered. The question of how the medium permeability affects the total record-reproduce process can only be answered when the record mechanisms are fully understood.

Appendix

The essence of any potential problem is expressed by the Green's function [6]. Given a region with a specified boundary and specified interior permeabilities, the Green's function is uniquely determined. Once known, it can be used to determine the field in the region, given specific boundary potentials (the field from a head) and/or interior charge density (the field due to the recorded magnetization patterns). The Green's function appropriate to the configuration shown in Fig. 1 is derived below.

In general, the constitutive relation (3) may be written as

$$B(r) = \mu_0(\tilde{\mu} \cdot H(r) + M_0(r)) \tag{A-1}$$

where H here represents the total field (demagnetization plus head field). If we write

$$H(r) = -\nabla V(r), \tag{A-2}$$

then the differential equation for V is

$$\nabla \cdot \tilde{\mu} \cdot \nabla V(r) = -\rho(r) \tag{A-3}$$

where the volume charge density $\rho(r)$ is

$$\rho(r) = -\nabla \cdot M_0(r). \tag{A-4}$$

The Green's function is the potential due to a point charge and thus satisfies

$$\nabla \cdot \tilde{\mu} \cdot \nabla G(r, r') = -\delta^3(r - r') \tag{A-5}$$

with $G(r, r')$ vanishing when one of its arguments concides with the region boundary (for the case of $V(r)$ specified on the region boundary).

Once the Green's function is determined, the potential may be calculated from

$$V(r) = \int_{\text{medium}} d^3 r' (M_0(r') \cdot \nabla') G(r, r') - \int_{\text{boundary}} d^2 r' V(r') \frac{\partial G}{\partial n}(r, r') \tag{A-6}$$

where $\partial G/\partial n$ is an outward normal gradient. (A-6) is appropriate for magnetization patterns (charge densities) in well-defined portions of the region. In the particular two-dimensional approximation for this analysis, the Fourier component of the potential is given by

$$v(k, y) = v(k, -a) \frac{\partial g(-a, y, k)}{\partial y'} + \int_0^d dy' \left[m_0(k, y') \cdot \left(-ik, \frac{\partial}{\partial y'} \right) \right] g(y', y, k) \tag{A-7}$$

where

$$g(y', y, k) = e^{ikx'} \int_{-\infty}^{\infty} e^{-ikx} \, dx \, G(x, y, x', y') \tag{A-8}$$

$v(k, -a)$ is the potential along the head surface ($y = -a$) and $(-ik, \partial/\partial y')$ is a vector. The Fourier component form for the field is

$$h(k, y) = -\left(ik, \frac{\partial}{\partial y} \right) v(k, y). \tag{A-9}$$

Utilization of (A-9) with (A-7) is straightforward. For the field per unit MMF from the head, m_0 is set to zero and the unit potential drop across the gap transforms to

$$v(k, -a) = i/k. \tag{A-10}$$

If the demagnetization (or fringing) fields from a magnetized medium are desired, $v(k, -a)$ is set to zero (which gives the imaging of the infinitely permeable head plane), and the integral part of (A-7) is utilized. To obtain the reproduce flux wavelength response utilizing (7), the field per unit MMF of the head is used which requires a knowledge of $g(y', y, k)$.

$g(y', y, k)$ is determined by Fourier transforming (A-5) for two dimensions:

$$\frac{\partial^2}{\partial y^2} g(y', y, k) - k^2 \frac{\mu_x}{\mu_y} g(y', y, k) = \frac{1}{\mu_y} \delta(y - y'). \tag{A-11}$$

Solving (A-11) with the usual surface flux and field continuity conditions yields the Green's function (A-12) where the pair (y, y') is assigned to $(y_<, y_>)$ depending on their relative magnitudes ($y_>$ is defined to be greater than $y_<$):

$0 < y_< < y_> < d$

$$g(y', y, k) = \frac{[\beta \cosh k\alpha(d - y_>) + \sinh k\alpha(d - y_>)]\ [\beta \tanh ka \cosh k\alpha y_< + \sinh k\alpha y_<]}{k\beta \cosh k\alpha d\, [\beta(1 + \tanh ka) + (1 + \beta^2 \tanh ka) \tanh k\alpha d]}$$

$0 < y_> < d \qquad -a < y_< < 0$

$$g(y', y, k) = \frac{[\beta \cosh k\alpha(d - y_>) + \sinh k\alpha(d - y_>)]\ \sinh k(y_< + a)}{k \cosh ka \cosh k\alpha d\, [\beta(1 + \tanh ka) + (1 + \beta^2 \tanh ka) \tanh k\alpha d]}$$

$-a < y_< < y_> < 0$

$$g(y', y, k) = \frac{(1 \mp \beta)\ [\beta \cosh k(\alpha d \pm y_>) + \sinh k(\alpha d \mp y_>)]\ \sinh k(y_< + a)}{2\, k \cosh ka \cosh k\alpha d\, [\beta(1 + \tanh ka) + (1 + \beta^2 \tanh ka) \tanh k\alpha d]} \tag{A-12}$$

(where a sum over the two sign complexions is inferred)

$$y_> > d$$

$$g(y', y, k) = g(d, y_<, k)\, e^{-ky_>}.$$

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References

[1] W. K. Westmijze, "Studies on magnetic recording," *Philips Research Report.*, 8, 245-269, 1953.

[2] D. L. Tjaden and A. M. Rijckaert, "Theory of anhysteretic contact duplication," *IEEE Trans. Mag.*, Mag-7, 532-536, 1971.

[3] T. Wessel-Berg and H. N. Bertram, "A generalized formula for induced magnetic flux in a play-back head," *IEEE Trans. Mag.*, to be published, May 1978.

[4] H. N. Bertram, "Wavelength response in AC biased recording," *IEEE Trans. Mag.*, Mag-11, 1176-1178, 1975.

[5] B. K. Middleton and P. L. Wisely, "The development and application of a simple model of digital magnetic recording to thick media," *IERE Conf. Proc. No. 35*, 33-42, 1976.

[6] J. D. Jackson, "Classicial electrodynamics," New York: John Wiley and Sons, 1962, pp. 18-20.